```
Clear["Global`*"]
```

This section is mostly not attempted. The problems seem too simplistic to challenge Mathematica. I feel that if difficult eigenvalue problems were to come up, the most efficient approach would likely be entirely different than anything referred to here. By the way, there is a recently discovered (Oct '18) bug in the Eigenvalues function , discussed at *https://-mathematica.stackexchange.com/questions/186224/wrong-eigenvalues-from-a-sparse-matrix-eigenvalues-are-nonreal*. It looks like the Arnoldi method may be at fault, and it might be a good idea to avoid it for now.

If it is necessary to numerically fish for eigenvalues, the **`QRDecomposition`** method, treated in section 20.9, is one to consider.

1 - 4 Power method without scaling
Apply the power method without scaling (3 steps), using $x_0 = \{1,1\}^T$ or $\{1, 1, 1\}^T$. Give Rayleigh quotients and error bounds.

1. $\begin{pmatrix} 9 & 4 \\ 4 & 3 \end{pmatrix}$

```
m1 = (9 4
      4 3)
```

```
{{9, 4}, {4, 3}}
```

```
Eigenvalues[m1]
```

```
{11, 1}
```

Just to blow out the cobwebs, let me try a couple of fairly large ones.

```
m30 = Table[RandomSample[Range[40], 30], {n, 1, 30}];
```

```
m30p = Table[m30[[n]] (-1)^m30[[n]], {n, 1, 30}];
```

Mathematica has no difficulty in finding eigenvalues of relatively sizeable matrices. Reviewing the suggested techniques in the documentation would serve better, probably, than trying out the limited approaches described in this section.

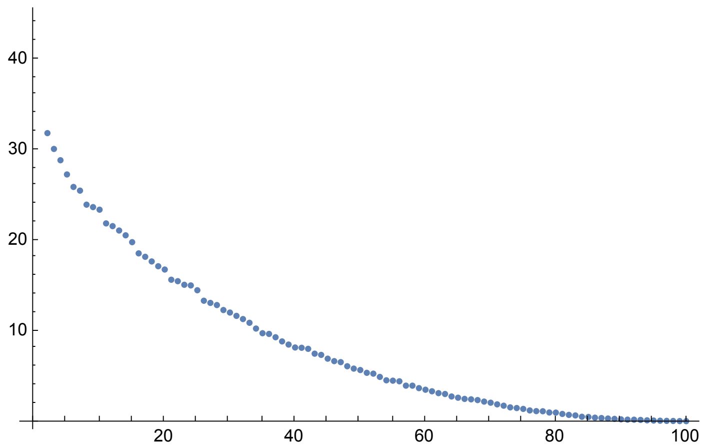
```
Timing[N[Eigenvalues[m30p]]]
```

```
{0.301897, {123.43 + 38.8128 i, 123.43 - 38.8128 i,
  -77.3667 + 93.2068 i, -77.3667 - 93.2068 i, 15.5139 + 118.638 i,
  15.5139 - 118.638 i, -107.456, 73.5353 + 73.4756 i,
  73.5353 - 73.4756 i, -82.818 + 49.6166 i, -82.818 - 49.6166 i,
  -45.9606 + 82.8799 i, -45.9606 - 82.8799 i, 94.5401, 43.3048 + 78.365 i,
  43.3048 - 78.365 i, -72.1173 + 21.2722 i, -72.1173 - 21.2722 i,
  66.5277 + 30.0296 i, 66.5277 - 30.0296 i, -13.1193 + 68.3912 i,
  -13.1193 - 68.3912 i, -46.1061 + 26.1212 i, -46.1061 - 26.1212 i,
  25.2435 + 39.9165 i, 25.2435 - 39.9165 i, 36.3336 + 16.044 i,
  36.3336 - 16.044 i, -12.9431 + 7.80501 i, -12.9431 - 7.80501 i}}
```

The eigenvalues of a random symmetric matrix:

```
r = RandomReal[1, {100, 100}];
r = Transpose[r].r;
vals = Timing[Eigenvalues[r]];
```

```
ListPlot[vals]
```



3. $\begin{pmatrix} 2 & -1 & 1 \\ -1 & 3 & 2 \\ 1 & 2 & 3 \end{pmatrix}$

```
Clear["Global`*"]
```

m1 = $\begin{pmatrix} 2 & -1 & 1 \\ -1 & 3 & 2 \\ 1 & 2 & 3 \end{pmatrix}$

```
{{2, -1, 1}, {-1, 3, 2}, {1, 2, 3}}
```

```
Eigenvalues[m1]
```

```
{5, 3, 0}
```

5 - 8 Power method with scaling

Apply the power method (3 steps) with scaling, using $x_0 = \{1, 1, 1\}^T$ or $\{1, 1, 1, 1\}^T$, as applicable. Give Raleigh quotients and error bounds.

5. The matrix in problem 3.

7. $\begin{pmatrix} 5 & 1 & 0 & 0 \\ 1 & 3 & 1 & 0 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 5 \end{pmatrix}$

```
m1 = (5 1 0 0
      1 3 1 0
      0 1 3 1
      0 0 1 5)
```

```
{{5, 1, 0, 0}, {1, 3, 1, 0}, {0, 1, 3, 1}, {0, 0, 1, 5}}
```

```
N[Eigenvalues[m1]]
```

```
{5.61803, 5.30278, 3.38197, 1.69722}
```

The above match the text answer to 4S.

9. Prove that if x is an eigenvector, the $\delta = 0$ in (2). Give two examples.

11. Spectral shift, smallest eigenvalue. In problem 3, set B = A - 3 i (as perhaps suggested by the diagonal entries) and see whether you may get a sequence of q's converging to an eigenvalue of A that is smallest (not largest) in absolute value. Use $x_0 = \{1, 1, 1\}^T$. Do 8 steps. Verify that A has the spectrum {0, 3, 5}.